Přednáška 04

Fyzikálně založené modelování odrazivosti

PBR: parametrické modely a jejich omezení

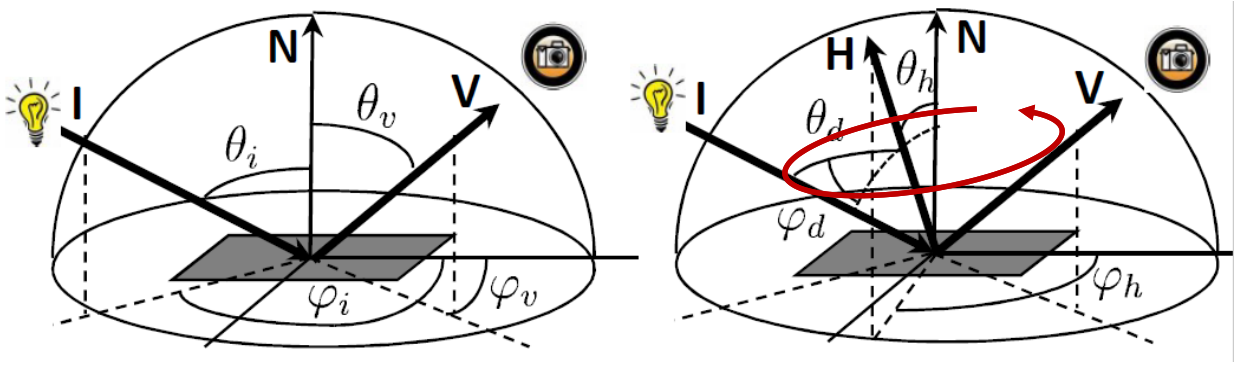
Jiří Filip filipj@utia.cas.cz

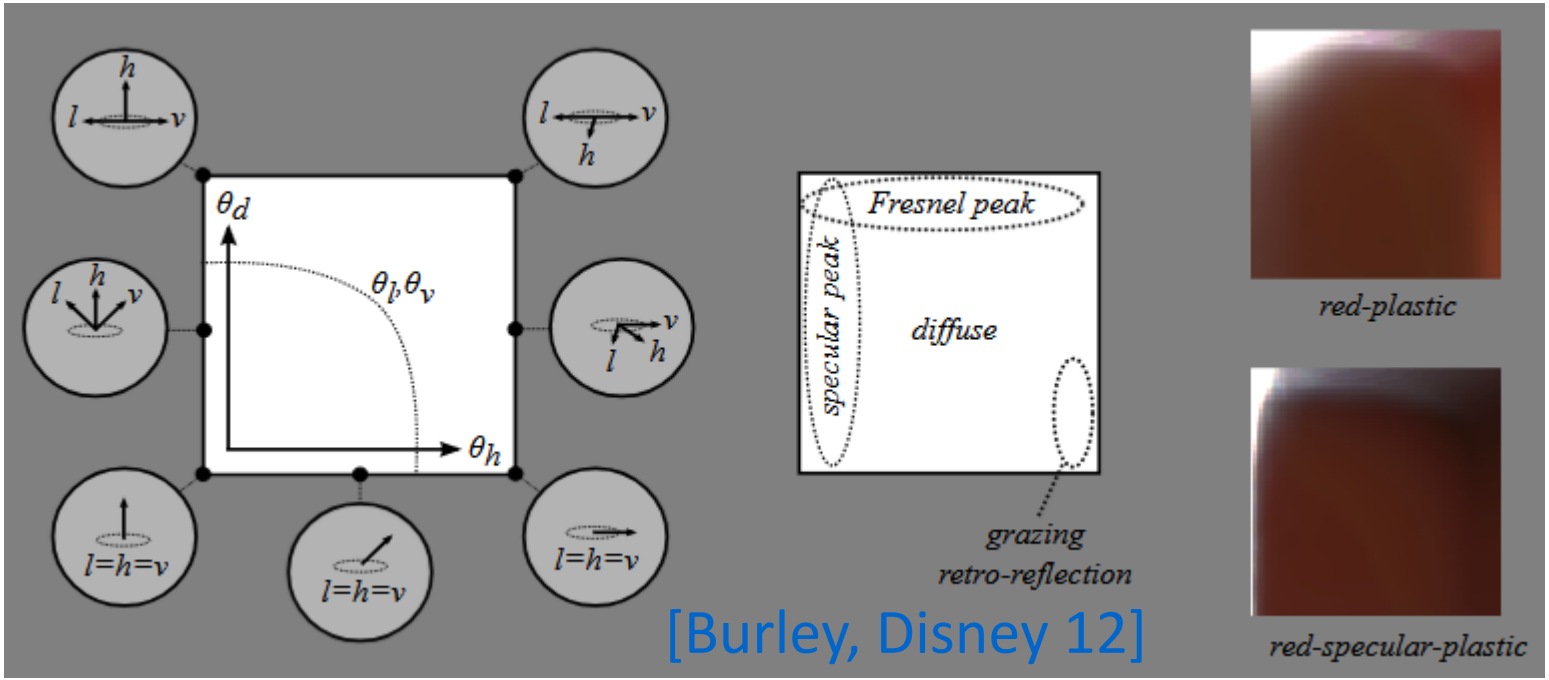
Czech Academy of Sciences



Half-Difference Parameterization

BRDF in half-difference parameterization [Rusinkiewicz 98]

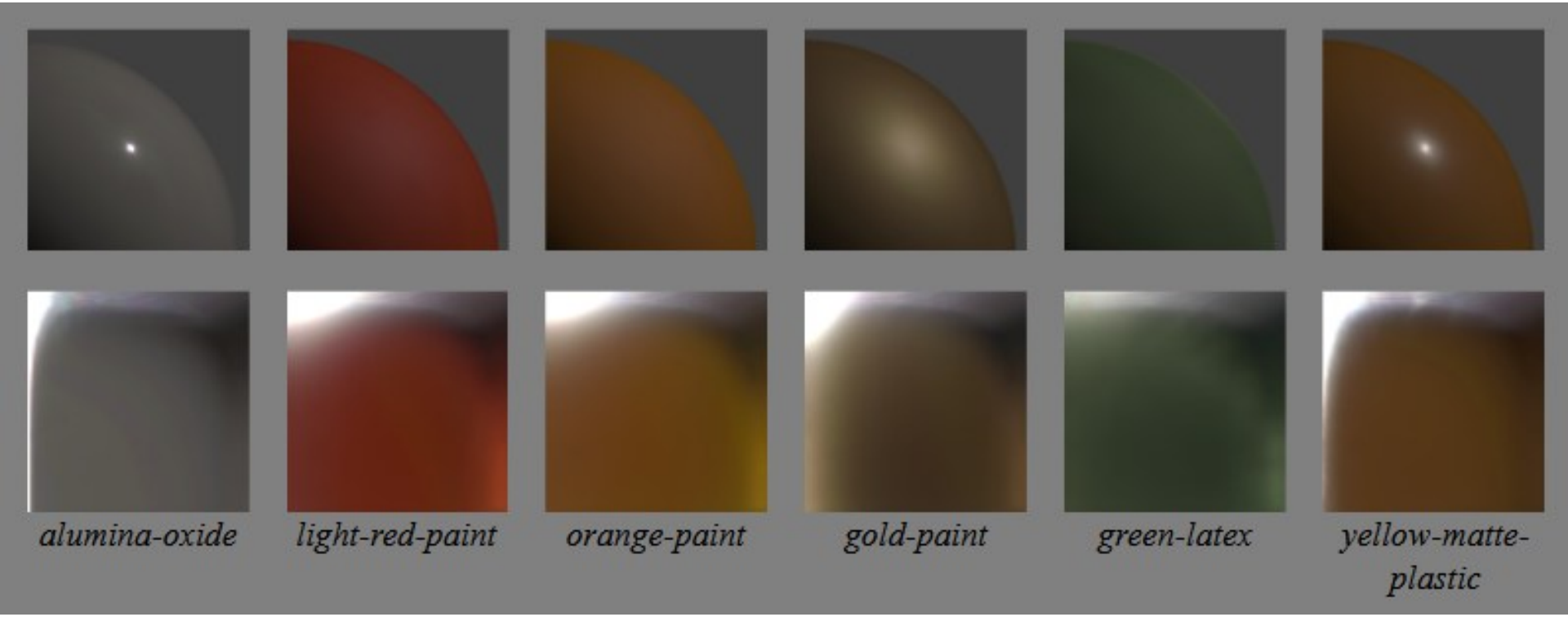




[Burley, Disney 12]

Half-Difference Parameterization

[Burley, Disney 12]



BRDF Data Compression

- **Splines** [He et al., SIG 92]
  - Used for storing precomputed BRDF model values
  - Do not exploit BRDF symmetry, low compression
- **Spherical harmonics** [Westin et al., SIG 92]
  - Analogy of sin,cos basis functions on sphere in frequency domain
  - Requires many parameters otherwise produces artifacts
- **Spherical wavelets** [Schroder & Sweldens, SIG 95]
  - Basis functions localized in both spatial and frequency domain
- **Zernike polynomials** [Koenderink et al., ECCV 96]
  - Polynomial functions used in optics as a basis functions mapped on hemisphere

BRDF Data Factorization

[Kautz et al. 99] – use ***SVD*** to produce two 2D factors instead of 4D BRDF.

$$BRDF(\omega_i, \omega_v) \approx \sum_{k=1}^{K_j} P_{k,r_1,r_2}(\pi_1(\omega_i, \omega_v)) Q_{k,r_1,r_2}(\pi_2(\omega_i, \omega_v))$$

[McCool et al. 01] – use ***Homomorphic factorization*** to generate more than two positive factors.

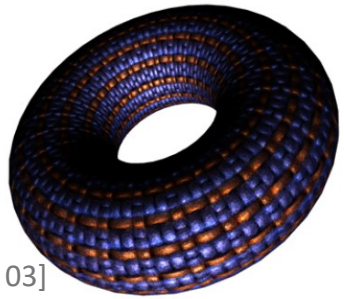
$$BRDF(\omega_i, \omega_v) \approx \prod_{j=1}^{J} P_{j,r_1,r_2}(\pi_j(\omega_i, \omega_v))$$

[Suykens et al. 03] – each pixel = product of three or more two-dimensional positive factors using ***chained matrix factorisation***.

$$BRDF(\omega_i, \omega_v) \approx \prod_{j=1}^{J} \sum_{k=1}^{K_j} P_{j,k,r_1,r_2}(\pi_{j,1}(\omega_i, \omega_v)) Q_{j,k,r_1,r_2}(\pi_{j,2}(\omega_i, \omega_v)$$

☑ factors in form of textures ⇨ interactive rendering

☒ For compression of measured BRDFs only



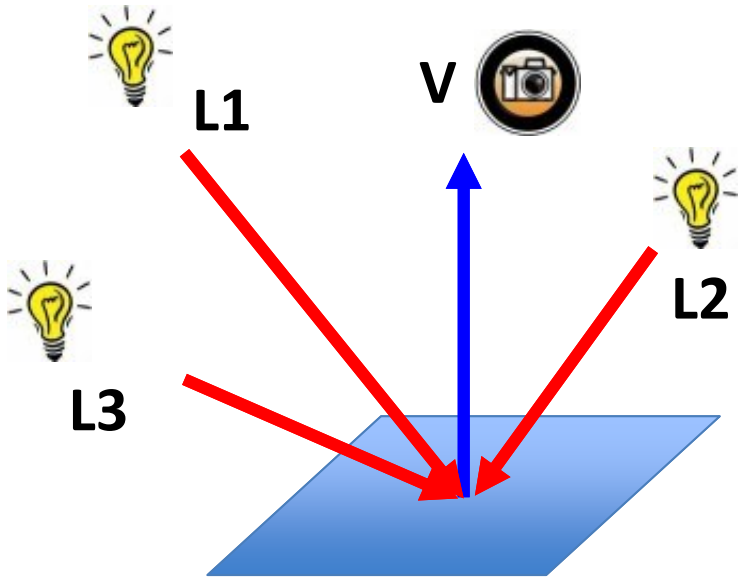
[Suykens et al. 03]

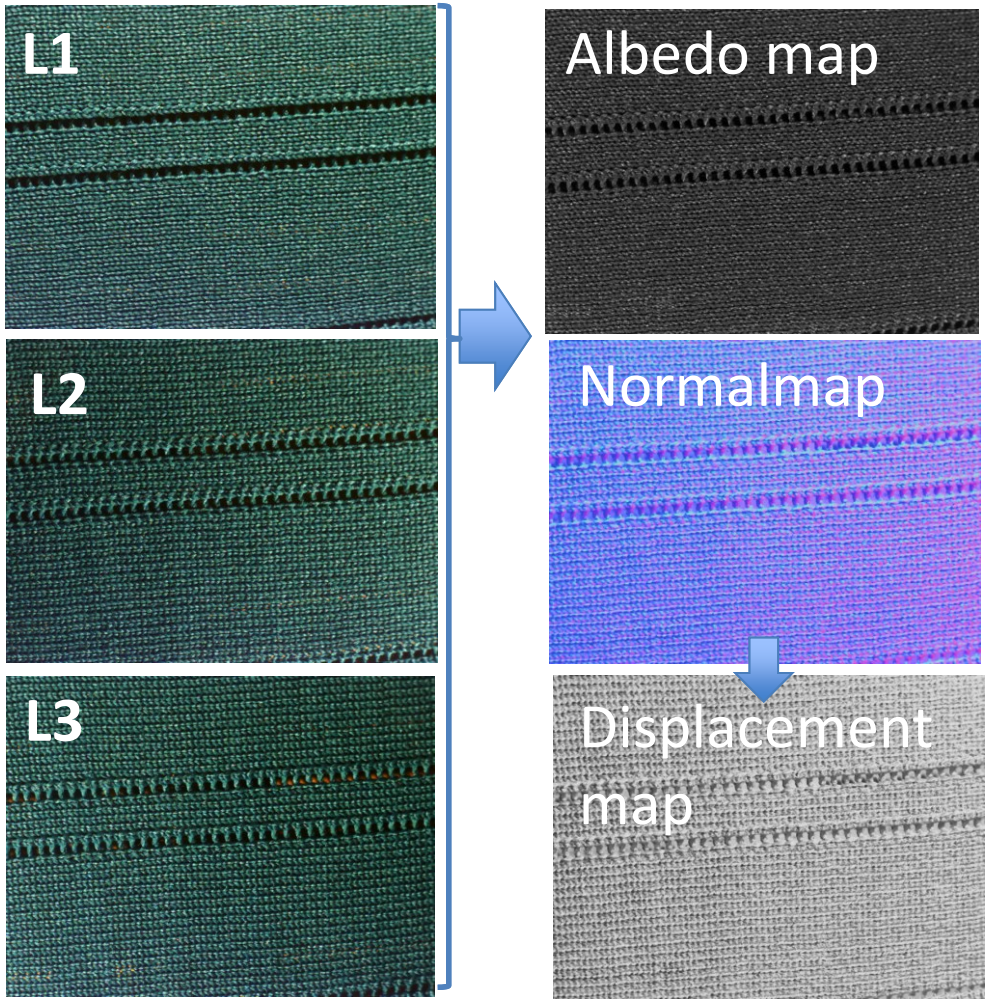
Photometric stereo (Lambertian surfaces)

[Frankot & Chellappa 88]

Shape-from-shading: one image → heavily ambiguous

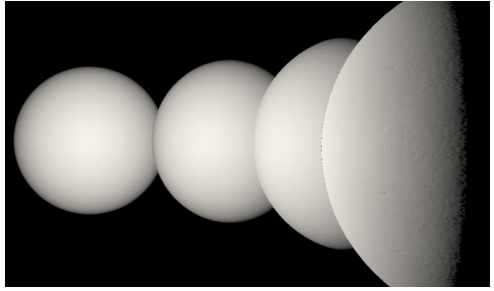
Photometric stereo: several known illuminations → ambiguity is strongly reduced



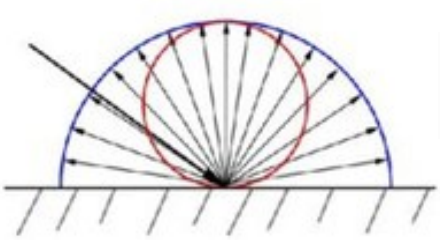


More views →
stereo geometry
More lights →
photometric stereo

Photometric stereo principle



For a Lambertian surface [Woodham 82]:



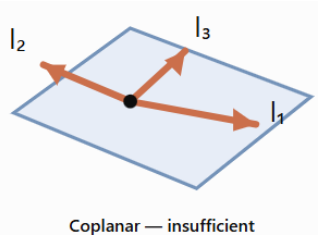
$$I_k = \rho\,(\mathbf{n} \cdot \mathbf{l}_k)$$

- I_k — measured intensity under light k
- ρ — diffuse albedo
- $\mathbf{n}$ — unknown unit surface normal
- $\mathbf{l}_k$ — known illumination direction

Scaled normal:

$$\mathbf{g} = \rho\mathbf{n}, \quad I_k = \mathbf{l}_k^T \mathbf{g}$$

- Fixed camera, varying illumination
- Pixel correspondence is automatic — geometry does not move
- Intensity changes encode **local surface orientation**
- At least **3 non-coplanar light directions** are required
- Output: **normal map + diffuse albedo map**
- Assumes approximately Lambertian reflectance and no shadows/inter-reflections
- **Optimal configuration:** azimuth separation 120°, elevation 54.7°



Calibrated photometric stereo

If illumination directions and intensities are known, measurements at each pixel can be written as:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \mathbf{l}_1^T \\ \mathbf{l}_2^T \\ \mathbf{l}_3^T \end{bmatrix} \mathbf{g} = \mathbf{L}\mathbf{g}.$$

For exactly three independent lights: $\mathbf{g} = \mathbf{L}^{-1}\mathbf{I}$

Albedo and normals: $\rho = \|\mathbf{g}\|, \qquad \mathbf{n} = \frac{\mathbf{g}}{\|\mathbf{g}\|}.$

- Light directions l_k are known
- Relative light intensities are known or normalized
- Camera response should ideally be linear/radiometrically calibrated
- Very simple per-pixel solution
- No explicit 3-D correspondence or stereo matching
- Captures **fine surface detail**
- Normals can be estimated at the full image resolution
- Albedo and orientation are separated

Overdetermined photometric stereo

In practice (BTF) **many more than three illumination directions**:

$$\mathbf{I} = \mathbf{L}\mathbf{g}, \qquad N > 3.$$

Estimate the scaled normal using least squares:

$$\hat{\mathbf{g}} = (\mathbf{L}^T\mathbf{L})^{-1}\mathbf{L}^T\mathbf{I} = \mathbf{L}^+\mathbf{I}.$$

Albedo and normals:

$$\rho = \|\hat{\mathbf{g}}\|, \qquad \mathbf{n} = \frac{\hat{\mathbf{g}}}{\|\hat{\mathbf{g}}\|}.$$

Why use more lights?

- Suppresses sensor noise
- Reduces sensitivity to individual measurements
- Better angular coverage
- Allows removal of measurements affected by shadows
- Allows detection/rejection of specular highlights
- More stable estimation for poorly oriented surfaces
- Provides redundancy for robust fitting

Robust and extended photometric stereo

The simple Lambertian model often fails because real materials exhibit:

$$I_k \neq \rho(\mathbf{n} \cdot \mathbf{l}_k)$$

- cast/self shadows
- specular highlights
- interreflections
- spatially varying reflectance
- non-Lambertian BRDFs

With many lights, these observations can be treated as **outliers**:

$$\hat{\mathbf{g}} = \arg\min_{\mathbf{g}} \sum_k w_k \left(I_k - \mathbf{l}_k^T \mathbf{g}\right)^2 .$$

<u>Photometric stereo provides</u>:

- Dense per-pixel normals
- Very high spatial detail
- Simple acquisition with a fixed camera
- Increasing robustness when many calibrated illuminations are available

Photmetric stereo methods overview

3-light solution	**Overdetermined solution**
Minimum acquisition	More images
Exact linear solution	Least-squares solution
Sensitive to noise	Noise averaging
Sensitive to one bad observation	Outlier rejection possible
Limited robustness	Much more practical

Typical extensions

- **Dark/bright samples filtering** — removing specular and shadowed directions
- **Weighted least squares** — downweight unreliable observations
- **Robust estimation / RANSAC** — reject highlights and shadows
- **Color photometric stereo** — exploit RGB channels
- **Near-field photometric stereo** — model spatially varying light direction and attenuation
- **Non-Lambertian PS** — explicitly model BRDF/specular reflection
- **Uncalibrated PS** — jointly estimate lighting and surface orientation
- **Deep photometric stereo** — learn mappings robust to complex reflectance

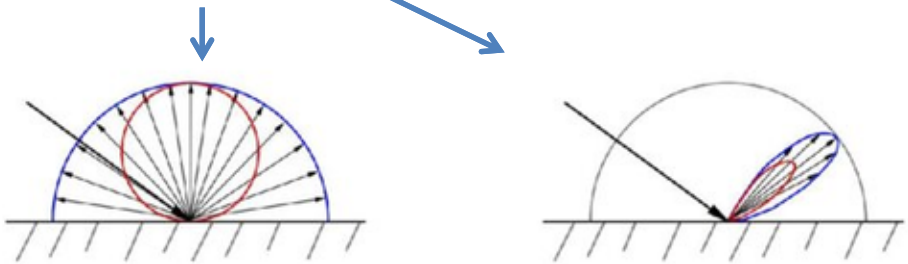
Principles of Analytical BRDF Modellling

- Incoming and outgoing directions

$$\mathbf{i}(\theta_i, \varphi_i) = [\sin\theta_i \cos\varphi_i, \sin\theta_i \sin\varphi_i, \cos\theta_i]$$
$$\mathbf{v}(\theta_v, \varphi_v) = [\sin\theta_v \cos\varphi_v, \sin\theta_v \sin\varphi_v, \cos\theta_v]$$

- Diffuse – specular linear decomposition

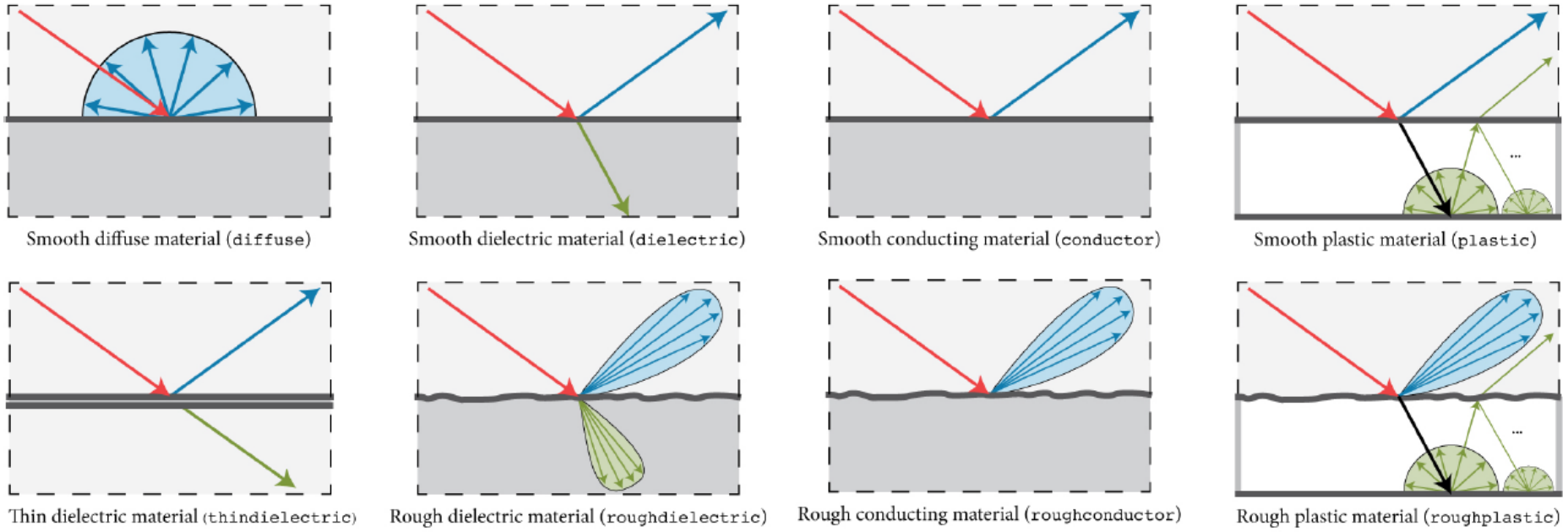
$$f(\mathbf{i}, \mathbf{v}) = \frac{k_d}{\pi} + k_s \cdot L(\mathbf{i}, \mathbf{v})$$



- Specular term example

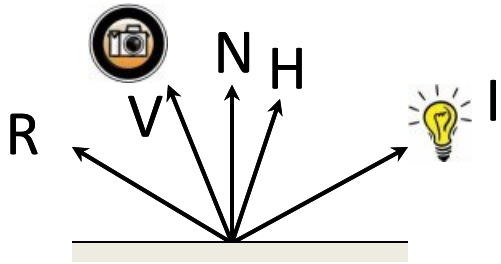
$$\frac{1}{\cos\theta_i \cos\theta_v} \frac{e^{-\tan^2\theta_h/\alpha^2}}{4\pi\alpha^2}$$

Principles of Analytical BRDF Modellling



Empirically Derived BRDF Models

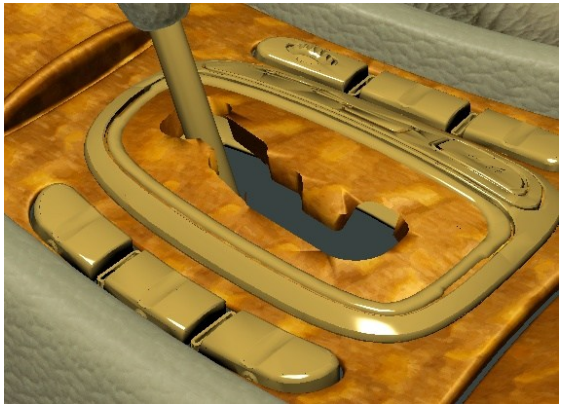
- **Phong shading** [Phong ACM 75]
  - Ambient, diffuse, and specular terms



$$BRDF(I,V) = k_a i_a + k_d (I \cdot N) i_d + k_s (R \cdot V)^{\alpha} i_s$$

$k \ldots$ material coefs., $i \ldots$ light coefs. $\quad R = 2(I \cdot N)N - I$

  - More computationally efficient modification [Blinn SIG 77] replaced term $R \cdot V$ by $N \cdot H$. Used in OpenGL and Direct3D implementations.
  - Improving energy conservation for metallic surfaces using facet-based model [Neumann et al. CGF 99].



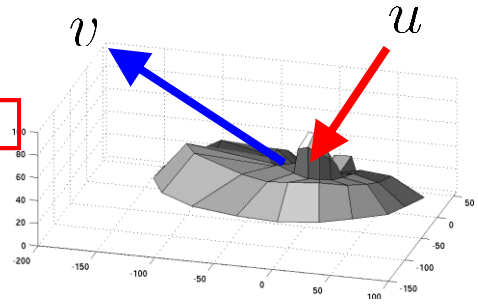
Empirically Derived BRDF Models

- [Schlick EG 94] - anisotropic, energy conserving, simplified Fresnel refraction $F(f_0,\theta) = f_0 + (1-f_0)(1-\cos\theta)^5$

GGX roughness shapes the highlight; Fresnel makes it stronger toward grazing angles and determines its intrinsic reflectance/color.

- [Lafortune et al. EG 97] generalized phys. plausible cosine lobes, one lobe model as 5 params.

$$Y_{i,v} = \rho[\omega_i^T \mathbf{D}\omega_v]^n = \rho(D_x u_x v_x + D_y u_y v_y + D_z u_z v_z)^n$$



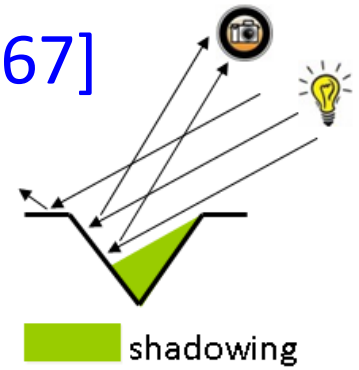
- Extension of Phong model

  [Ashikhmin & Shirley JGT 00]

  - non-Lambertian diffuse term, anisotropic, energy conserving, Fresnel refraction
  - intuitive parameters, complex computation

Physically-Derived BRDF Models

- **Micro-facet models** [Torrance & Sparrow JOSA 67]
  - Diffuse (Lambertial lobe) and scattering parts
  - Each facet – long V-cavity ⇨ perfect reflector
  - Random sizes and Gaussian distribution
  - Improvement [Cook & Torrance SIG 81], reflectance as
    - Fresnel function *F*
    - Facet distribution *D*
    - Shadowing/Masking term *G*



$$BRDF(\theta_i, \theta_v) = \frac{F(\theta_h)D(\theta_h)G(\theta_i, \theta_v)}{\pi \cos\theta_i \cos\theta_v}$$

D: where and how broad the highlight is,
G: masking and shadowing of microfacets,
F: how strongly those microfacets reflect depending on angle and material.

  - Complete model [He et al. CG 91]
    - Inter-reflections, occlusions, polarization, interference, diffraction, wave effects of light, …

☒ Isotropic reflections only

Physically-Derived BRDF Models

- Simplified analytical microfacet model [Ward CG 92]
  - Specularity as exp() function, four physically meaningful parameters, anisotropy modeling

$$BRDF(\theta_i, \theta_v) = \frac{k_d}{\pi} + k_s \frac{1}{\cos\theta_i \cos\theta_v} \frac{e^{-\tan^2\theta_h/\alpha^2}}{4\pi\alpha^2}$$

  - normalization in [Duer 05]
  - Anisotropic variant – specular term:

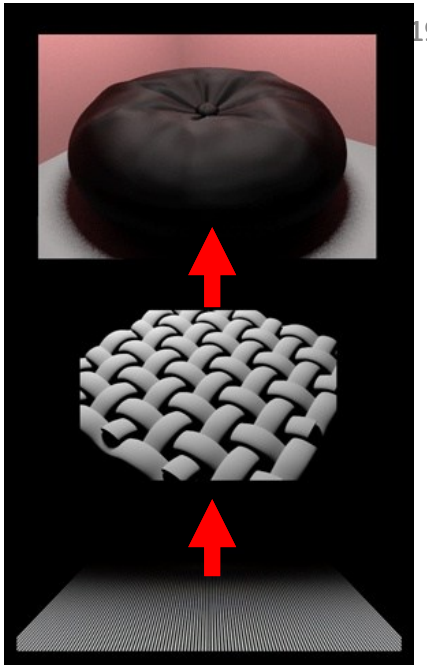
$$L_{Ward}(\mathbf{i}, \mathbf{v}) = \frac{F(f_0, \langle \mathbf{v} \cdot \mathbf{h} \rangle)}{\pi \alpha_x \alpha_y} \frac{1}{4\langle \mathbf{i} \cdot \mathbf{h} \rangle^2 h_z^4} \exp^{-\frac{1}{h_z^2}\left(\frac{h_x^2}{\alpha_x^2} + \frac{h_y^2}{\alpha_y^2}\right)}$$

Physically-Derived BRDF Models

- Model of diffuse reflection from rough surfaces

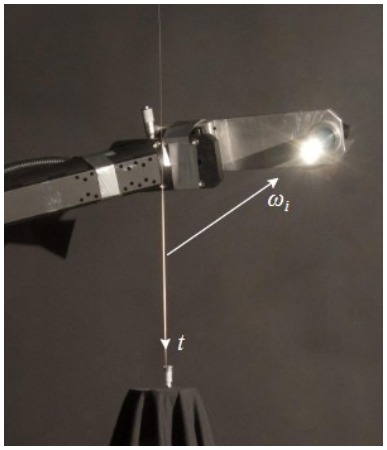
  [Oren & Nayar IJCV 95]
  - Uses [Torrance & Sparrow JOSA 67] micro-facet model,
  - Roughness as probability distribution of facet slopes,
  - Each facet has Lambertian reflectance.
- [Schlick 94 CGF 94]
  - Anisotropic, Sub-surface effects in layered materials, energy conservation
  - Account for difference between homogeneous and heterogeneous materials
  - Variable complexity formulations

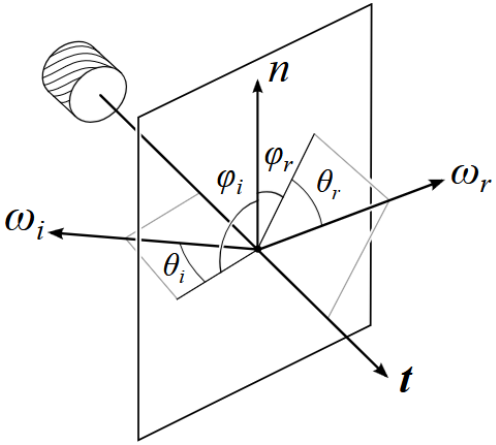
Microgeometry BRDF Models

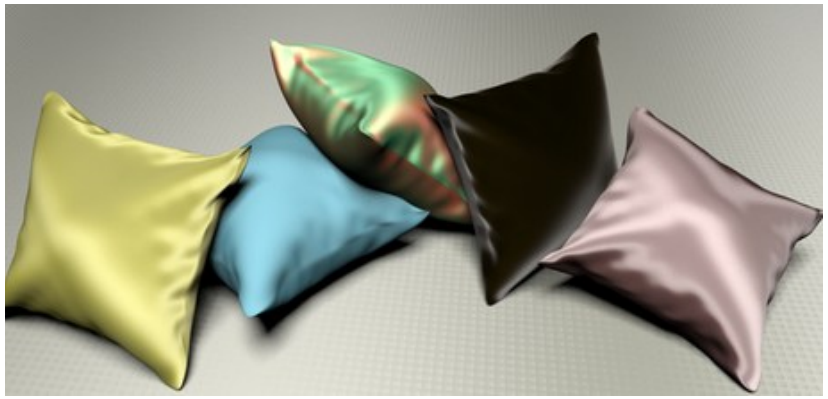


- Microgeometry model [Westin 92]
  - Geometry based model ⇨ More general
  - ☒ Underlying material geometry has to be known, difficult to fit to measured BRDFs

- BSDF measurement of threads combined with weaving pattern [Sadeghi 2013], [Xia 2025]
  - Microcylinder model (surface reflection, volume scattering)







Physically-Derived BRDF Models

- [Kurt et al. CG 10] – modification of Cook-Torrance microfacet model.
  - Anisotropic extension of facet distribution, energy conservation, simple fitting, fast rendering

Cook-Torrance:

$$BRDF(\theta_i, \theta_v) = \frac{k_d}{\pi} + k_s \frac{F(\theta_h) D(\theta_h) G(\theta_i, \theta_v)}{\pi \cos\theta_i \cos\theta_v}$$

Kurt et al.:

$$BRDF(\theta_i, \theta_v) = \frac{k_d}{\pi} + k_s \frac{F(\mathbf{VH}) D(\theta_h, \varphi_h)}{4(\mathbf{VH})(\cos\theta_i \cos\theta_v)^{\alpha}}$$

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Physically-Derived BRDF Models

- **GGX model** [Walter07], Kavoosighafi22]
  - modification of Cook-Torrance microfacet model
  - surface consists of many microscopic mirror-like facets with a statistical distribution of orientations

$$L_{GGX}(\mathbf{i},\mathbf{v}) = \frac{F(f_0,\langle \mathbf{v}\cdot\mathbf{h}\rangle)D(\mathbf{h})G(\mathbf{i},\mathbf{v})}{4\cos\theta_i\cos\theta_v}$$

$$\mathbf{h} = \frac{\mathbf{i}+\mathbf{v}}{||\mathbf{i}+\mathbf{v}||} \qquad \mathbf{h} = [h_x, h_y, h_z]$$

Fresnel term

$$F(f_0,\theta) = f_0 + (1-f_0)(1-\cos\theta)^5$$

Normal Distribution Function (NDF)

$$D(\mathbf{h}) = \frac{1}{\pi\alpha_x\alpha_y\left(\frac{h_x^2}{\sigma_x^2}+\frac{h_y^2}{\sigma_y^2}+h_z^2\right)^2}$$

Geometry/masking-shadowing

$$G(\mathbf{i},\mathbf{v}) = \frac{1}{(1+\Lambda(\mathbf{i}))(1+\Lambda(\mathbf{v}))}$$

$$\Lambda(\mathbf{v}) = \frac{-1+\sqrt{1+\frac{\alpha_x^2 v_x^2+\alpha_y^2 v_y^2}{v_z^2}}}{2}$$

Disney Principled BRDF

- **Goal:** one intuitive model for a wide range of real-world materials [*Burley, Disney,* 12]

$$f_{\text{Disney}} \approx f_{\text{diffuse}} + f_{\text{GGX specular}} + f_{\text{clearcoat}} + f_{\text{sheen}}$$

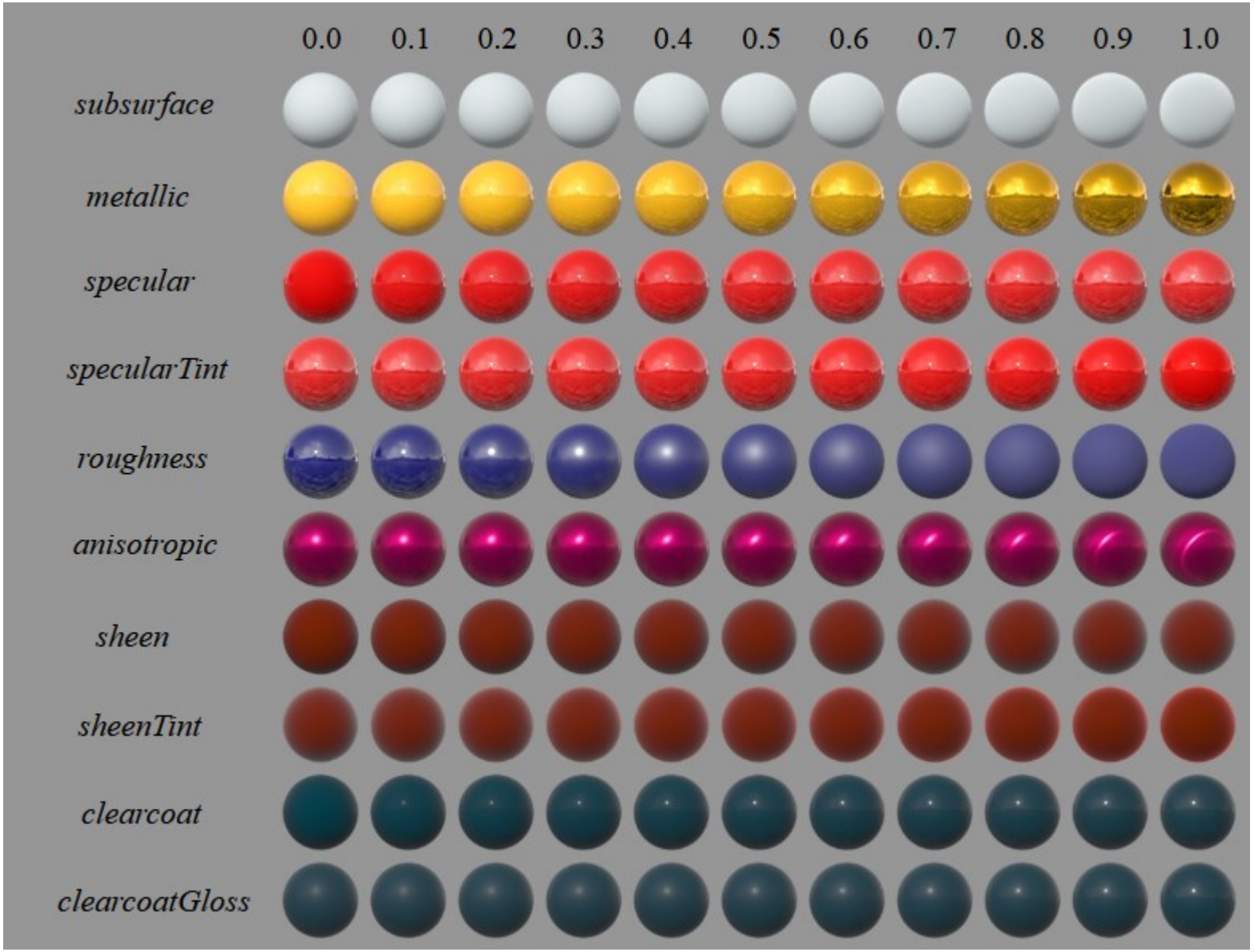
Main components

- **Diffuse:** modified Lambertian response, including grazing-angle behavior
- **Specular:** microfacet reflection using **GTR2 / GGX-like distribution**
- **Sheen:** soft grazing reflection, useful especially for cloth
- **Clearcoat:** additional narrow glossy layer, based on **GTR1**

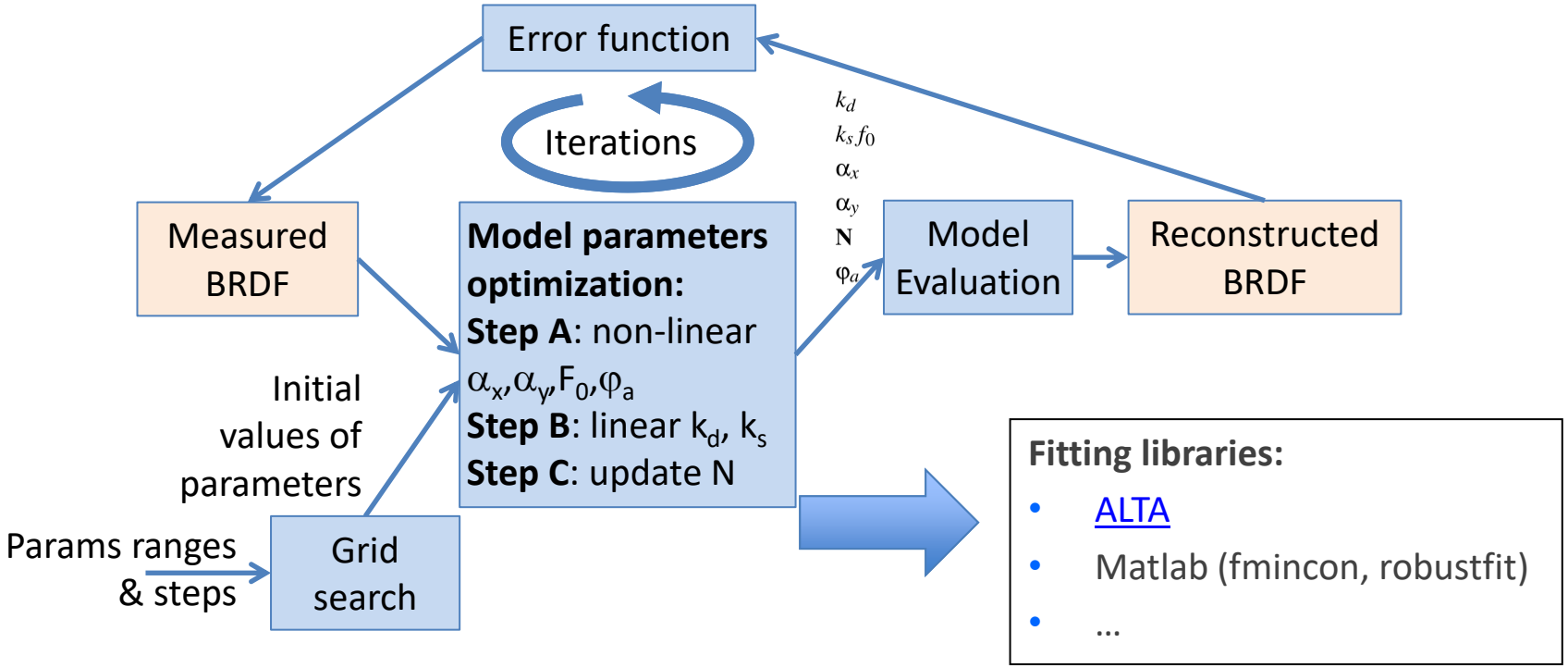
Artist-friendly parameters

- **Base Color** — intrinsic material color
- **Metallic** — dielectric <-> metal transition
- **Roughness** — width of the main specular highlight
- **Specular** — dielectric F_0 / specular strength
- **Anisotropic** — directional highlight shape
- **Sheen** — cloth-like grazing reflection
- **Clearcoat / Clearcoat Gloss** — secondary glossy coating

Disney Principled BRDF

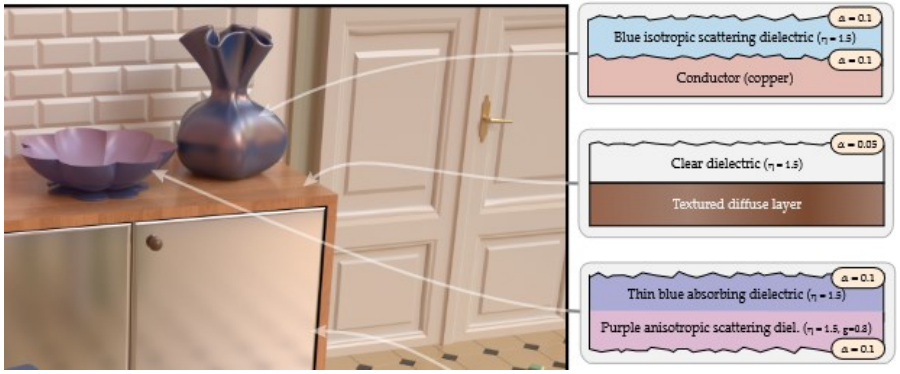


BRDF model parameters fitting



- Tricky part: **parameters initialization** (best selection using a grid search)
- Crucial component: **error function** evaluating fitting quality
- Often **time consuming** especially for multi-lobe fits
- Fitting **stability issues** for some pixels -> pooling surrounding pixels for parameters initialization

Layered BRDF / BSDF Models

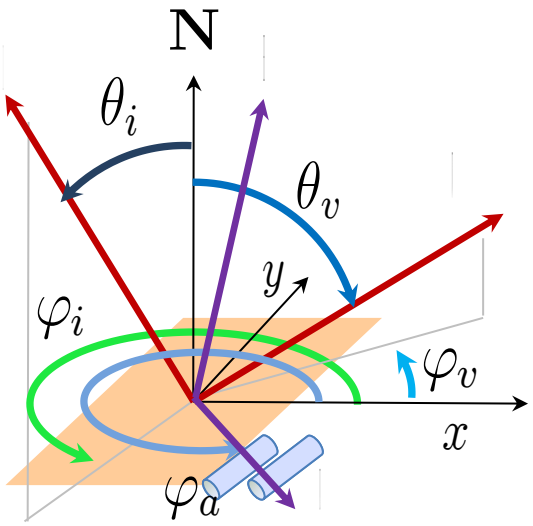


[Jacob et al. 14]

- **Appearance emerges from repeated light transport through interfaces and scattering layers:** e.g. clear-coated paint, glazed ceramics, varnished wood, skin, leaves, metallic coatings.
- Each layer or interface contributes different scattering behavior.
- A material is represented as a **stack of interfaces and participating layers**, e.g.: air→rough dielectric→scattering/absorbing layer→substrate.
- Individual components can describe **smooth or rough dielectric/conductor interfaces** and **scattering or absorbing media**.
- Light can undergo **multiple reflection, transmission, scattering and absorption** between layers → the resulting BRDF is therefore not simply a sum of independent lobes. Fresnel reflection at interfaces determines how much light is **reflected vs. transmitted** into deeper layers.

- **Key advantage:** complex material appearance emerges from physically meaningful layer parameters instead of requiring a specially designed BRDF for every material.
- **Limitation:** the basic formulation assumes plane-parallel layers and neglects lateral transport, so it produces a **BSDF rather than a spatially varying BSSRDF**.

Anisotropic highlights prediction

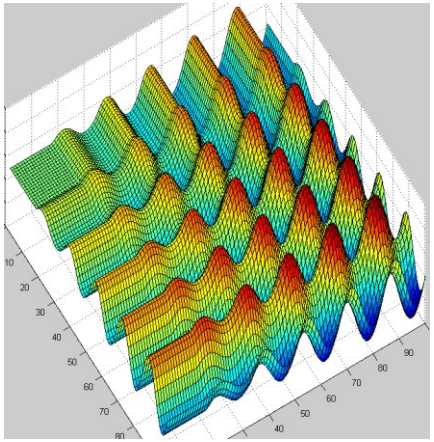


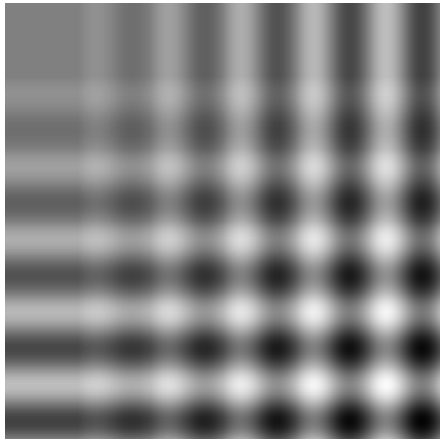
[Lu00CVIU] elements contributing to anisotropic reflection lie in planes orthogonal to half-way direction

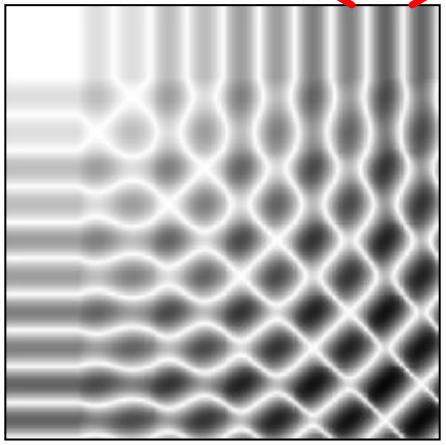


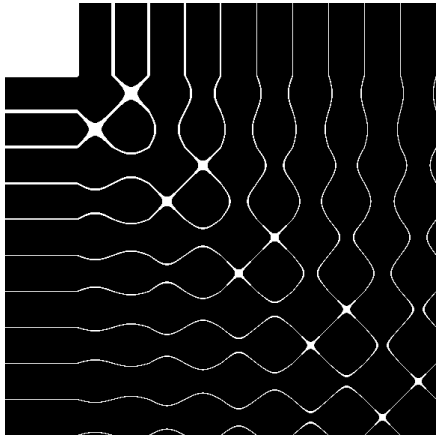
… direction of maximum material's anisotropy

Anisotropic stencil

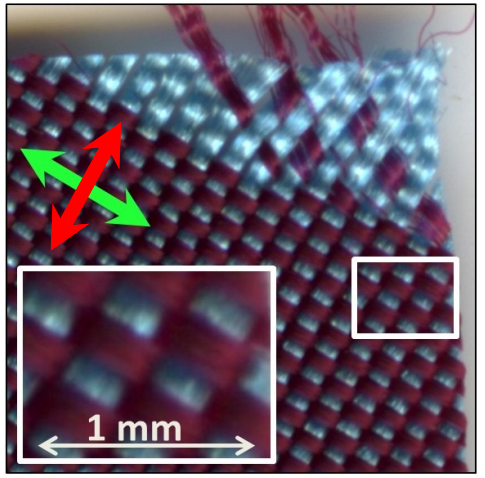




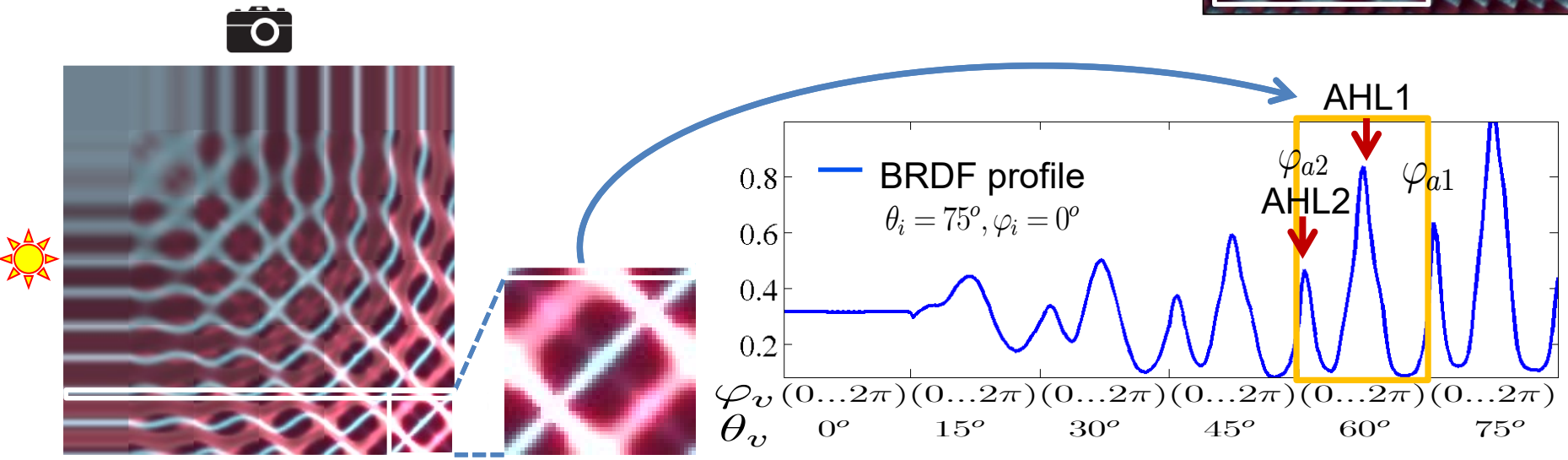




Anisotropy axes detection



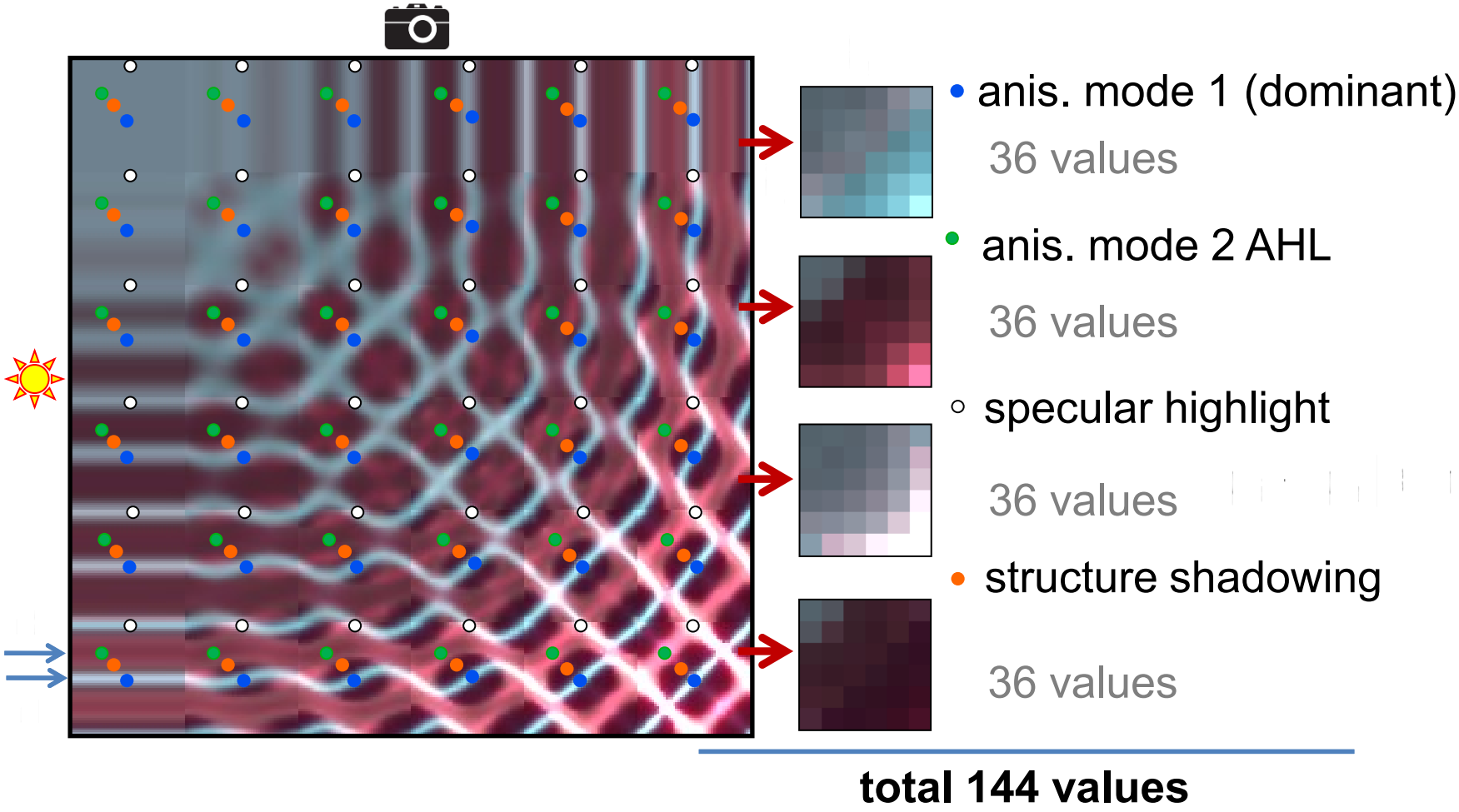
- Azimuthal location of anisotropy modes can be estimated automatically (2 modes)



- **Atypical parameters fitting**
  Instead of global optimization ⇨ individual anisotropy modes are fitted independently ⇨ finally their contributions combined

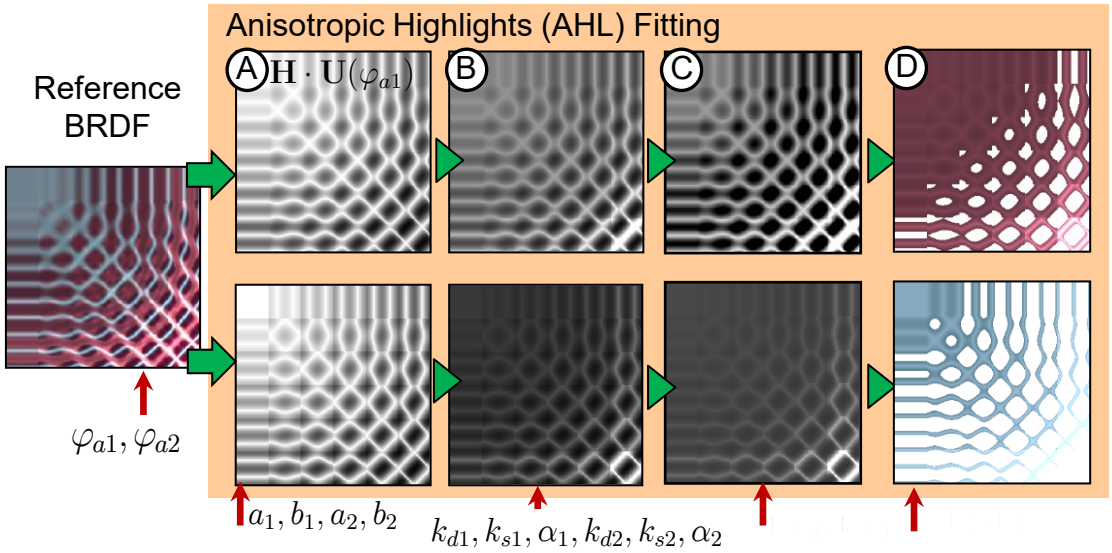
BRDF Data Sampling

- Ideally near uniform BRDF sampling



- Adaptation to non-uniform samples possible

BRDF Model Overview

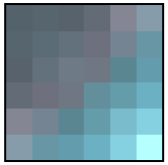


Ⓐ **Predicting anisotropic stencils (highlights locations)**

Ⓑ **Adding illumination/view elevation dependency**







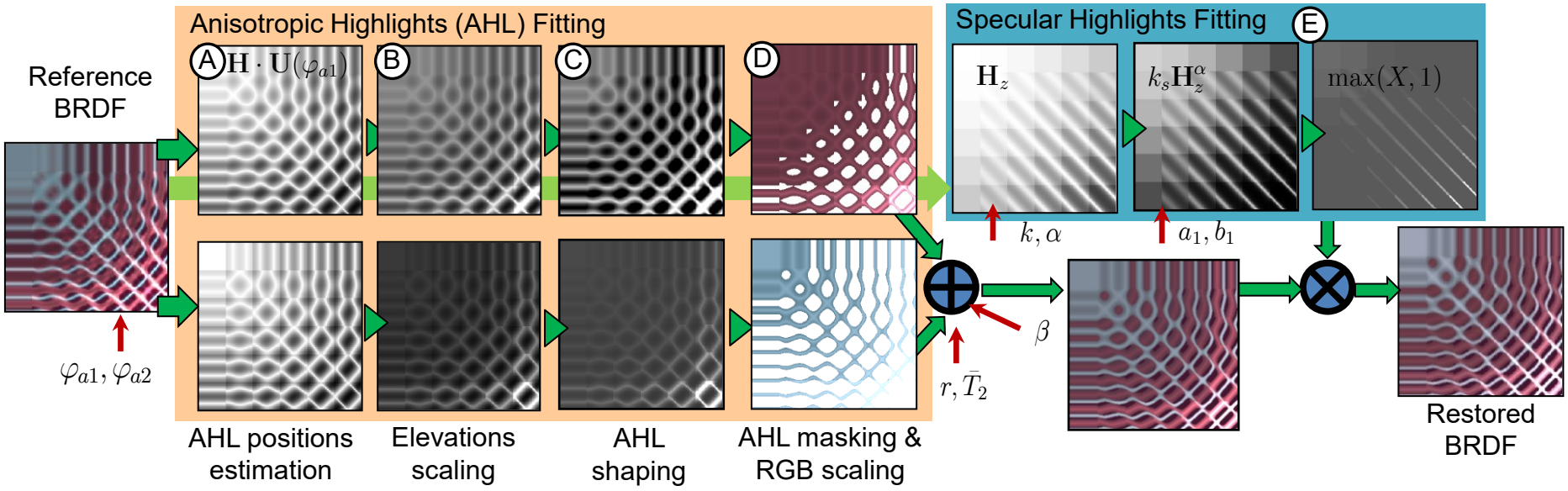
Ⓒ **Shaping highlights using Blinn model + RGB scaling**

… max() assures BRDF non-negativity

Ⓓ **Thread support masks computation**

… masks are needed for a correct threads contribution combination

BRDF Model Overview



Model equation (single anisotropy mode)



Combining anisotropy modes

… individual threads

$$B_i = M_{\varphi_{a1}}(w_1) T_{1i} + M_{\varphi_{a2}}(w_2) T_{2i}$$

… threads overlap

… background

Ⓔ Adding specular highlights



highlight shaping

elevation dependency

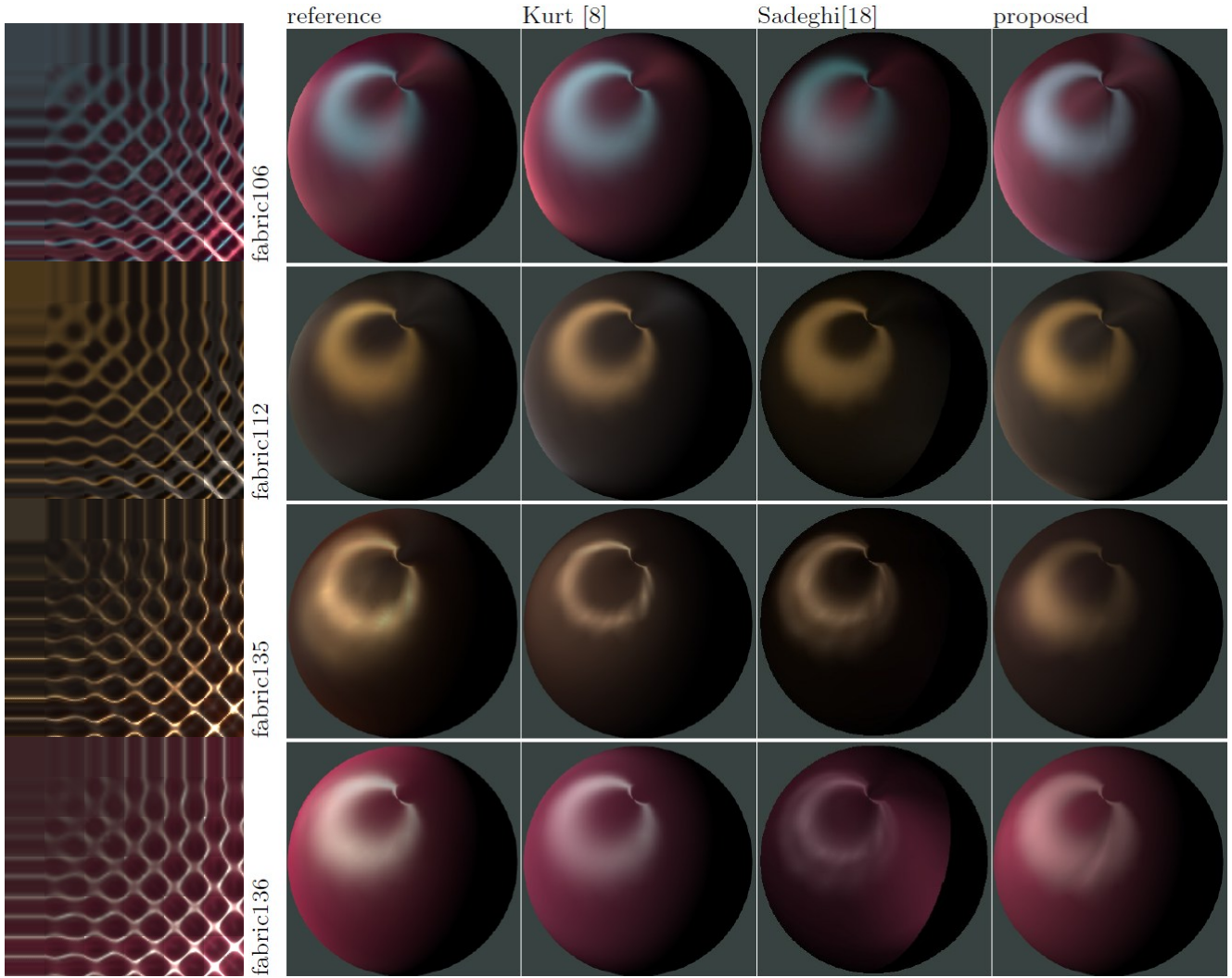
Parameters Fitting

- Model has 10 parameters per anisotropy mode
  $\varphi_a, a, b, k_d, k_s, a, m_r, m_g, m_b, w$
- Additional 4 global parameters:
- Fast parameters fitting using MATLAB:
  - `fit()` – elevations scaling
  - `nlinfit()` – Blinn model fitting
  - least squares fit
- Very fast ⇨ **~3s**
- Minimal number of samples for fitting ~144 but more is needed for optimal AHL shaping
- All presented results obtained by automatic fitting +
- Implementation of our model is available at:

  http://staff.utia.cz/filip/resources/stencils.zip

Results – point-light illumination

- Comparison with two anisotropic models

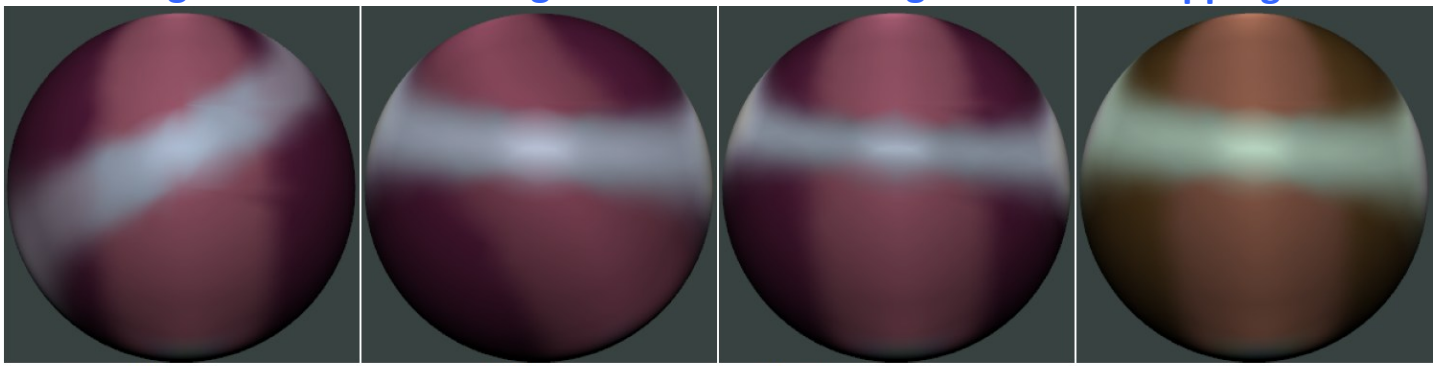


Point-light illumination

Tangents along sphere parallels

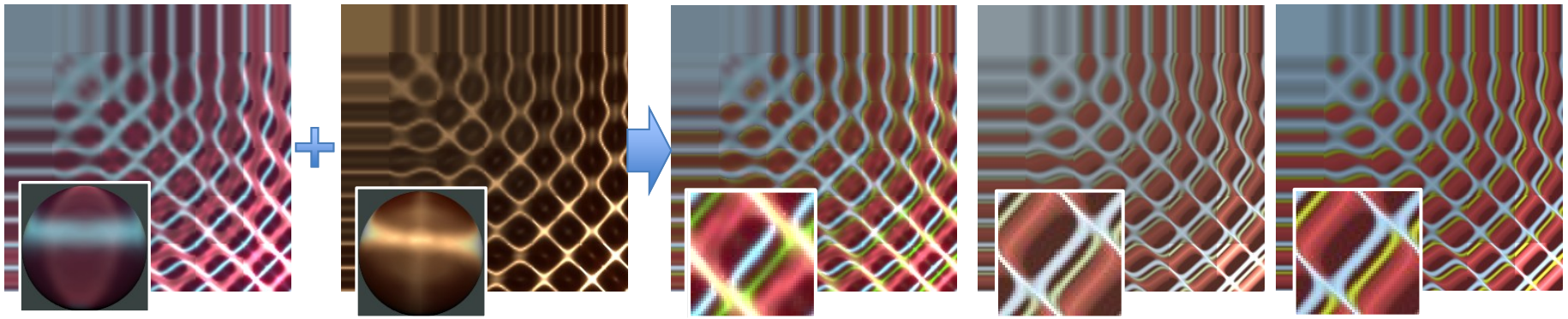
Anisotropy Editing

- Anisotropic highlights editing

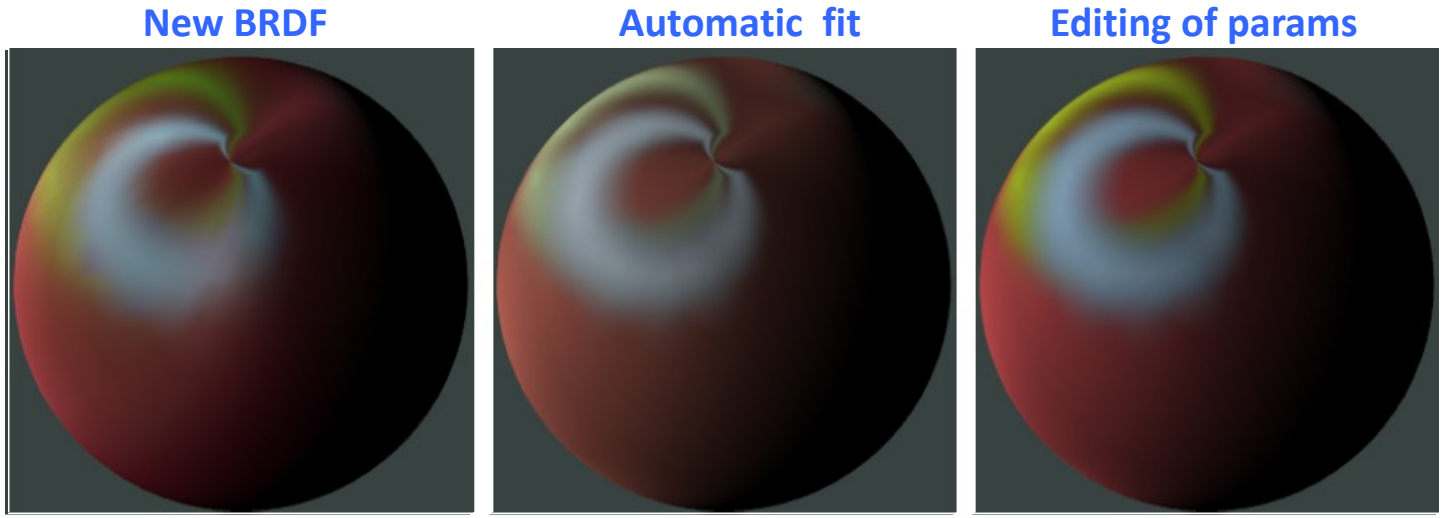


Modelling multiple anisotropy modes

- Fabricating 3-axes anisotropy



- Modelling of 3-axes anisotropy



GPU Implementation

- OpenGL GPU implementation of the model (2 axes)
  - 10k polygons, 800x600 ~**40-70 FPS**

```
vec4 BRDFstencils(in vec3 lightDir, in vec3 cameraDir, float theta_i, in float theta_v)
{
  vec3 H = normalize((lightDir + cameraDir)/2.0);
  float costicostv = cos(theta_i)*cos(theta_v);
  float dw = 0.1; // blending area

  // THREAD 1 ========================= (fabric106 blue)
  vec3 U1 = vec3(sin(adir1),cos(adir1),0.0);
  float F1 = 1.0-abs(dot(H,U1));
  float el1 = l1.x + l1.y/costicostv;
  float E1 = F1*el1;
  float bm1 = max(0.0,(bl1.x+bl1.y*pow(E1,bl1.z)));
  vec3 T1 = k1*bm1;
  float M1 = smoothstep(w1-dw,w1+dw,F1);

  // THREAD 2 ========================= (fabric106 red)
  vec3 U2 = vec3(sin(adir2),cos(adir2),0.0);
  float F2 = 1.0-abs(dot(H,U2));
  float el2 = l2.x + l2.y/costicostv;
  float E2 = F2*el2;
  float bm2 = max(0.0,bl2.x+bl2.y*pow(E2,bl2.z));
  vec3 T2 = k2*bm2;
  float M2 = smoothstep(w2-dw,w2+dw,F2);

  // combine BACKGROUND + THREAD1 + THREAD2 =================
  vec3 B = bcg;
  if(M2>0.0)
  B = B+M2*T2;
  if(M1>0.0 && M1<1.0)
  B = (1.0-M1)*B+M1*T1;
  if(M1==1.0)
  B = M1*T1;
  if(M1==1.0 && M2==1.0)
  B = blend*M2*T2 + (1.0-blend)*M1*T1;

  // specular part ============================================
  float S = max(1.0, spec.x*pow(H.z,spec.y)*el1 );

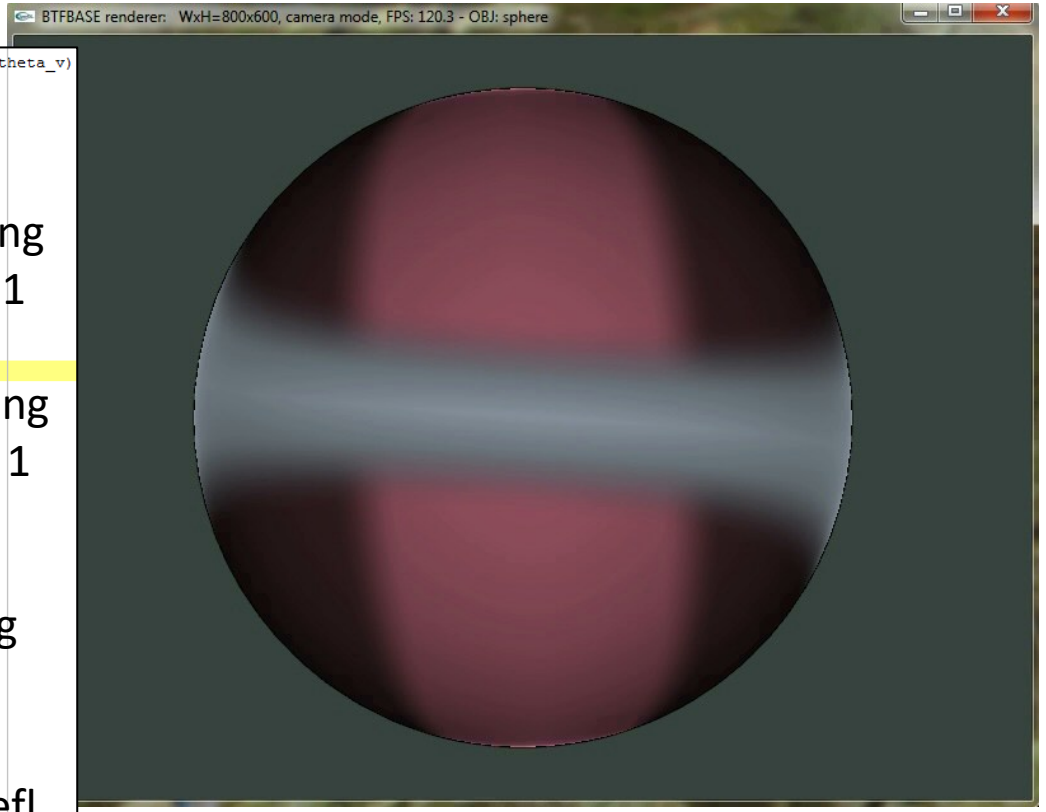
  vec4 resRGB = vec4(B*S*cos(theta_i),0.0);
  return resRGB;
}//--- BRDFstencils ---------------------------------------
```

Restoring thread 1

Restoring thread 1

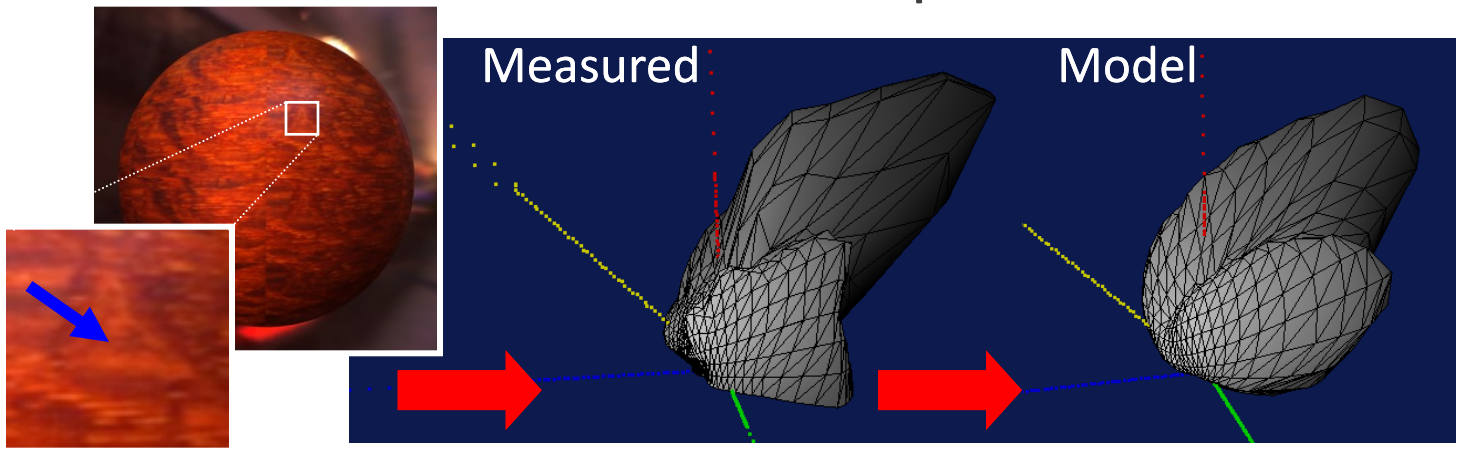
Combining threads

Specular refl.



Conclusions on BRDF Modeling

- Wide range of BRDF modeling and compression techniques available
  - Non-linear iterative estimation of parameters, depends on initialization
  - Memory efficient representation of BRDF
- Results of BRDF models ⇨ low-pass filter.



- Higher quality ⇨ more parameters to store
  ⇨ often more complex fitting

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